APPLICATION OF A NONLINEAR FRACTURE PROCESS ZONE MODEL TO AVALANCHE INITIATION AND IMPLICATIONS FOR FIELD TESTS

Thomas J. Boone[1]*

[1] *T.J. Boone Geotechnical and Reservoir Consulting, Canmore, AB, Canada*

ABSTRACT: Concepts have been developed in the study of fracture mechanics of rock that provide a critical link between results of tensile tests, fracture energy or fracture toughness and the nonlinear fracture process zone length behind the crack tip. This paper addresses how these same concepts can be extended and applied to weak layers in snow packs that are often associated with avalanches. The nonlinear fracture mechanics model described in this paper bridges this gap allowing for modelling of fracture initiation and propagation at these smaller scales. The model also provides a direct correlation between the results of a controlled compression test, the fracture energy associated with the weak layer and global instability of the snow pack. The resulting insights are found to be largely consistent with field experience utilizing hand compression tests.

KEYWORDS: avalanche initiation, avalanche mechanics, weak layer, slab, compression test

1. INTRODUCTION

This paper applies concepts developed in the study of the fracture mechanics of rock to snow avalanche mechanics and prediction. A simplified model for avalanche initiation mechanics is described where a buried weak layer with nonlinear material failure model is overlain and underlain by snow slabs that are assumed to be elastic materials. The process of initiating a slab avalanche can be summarized as shown in Figure 1 (Schweizer et al. 2003, McClung and Schaerer 2006). First, a load such as a skier causes a local failure of the weak layer. Second, if the snow pack/weak layer system is unstable, a compressive failure, anti-crack or fracture will propagate within the weak layer away from the local failure. And finally, if the gravity forces acting on an inclined slab are sufficient to overcome the newly frictional material resistance of the failed weak layer, the slab will begin to slide as an avalanche.

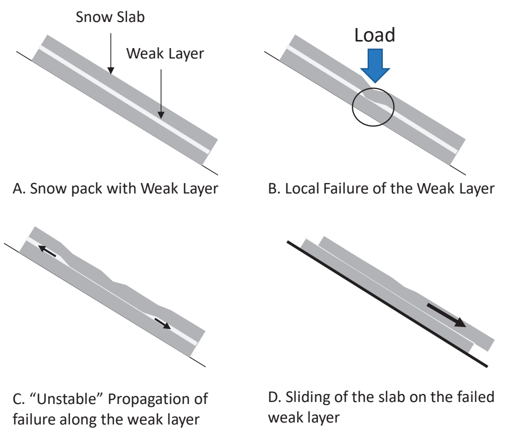


Figure 1. Simplified mechanics of an initiation and propagation of a slab avalanche on a weak layer in a snow pack.

This paper and the model described herein focuses on the initiation and propagation of the compressive failure. It should be noted that this ‘unstable’ progressive propagation of failure in the weak layer can occur in flat areas or on slopes so it doesn’t necessarily result in a sliding snow slab. Furthermore, the area and extent of a sliding slab will typically be different than the area of the initial compressive fracture.

In the field, these propagating compressive fractures are observed as “whumpfs”. Whumpfs can be interpreted as the vertical collapse of the weak layer from a state of high porosity to a state of normal consolidation or porosity over a significant area after unstable propagation of failure in the weak layer. To simplify the analysis in the paper, only a horizontal weak layer is considered and the time varying properties of snow are not considered.

2. LINEAR ELASTIC FRACTURE MECHANICS

Several authors have applied Linear Elastic Fracture Mechanics (LEFM) to the problem of initiation of slab avalanches (Heierli et al. 2008). However, in order to apply Linear Elastic Fracture Mechanics (LEFM) to any fracture, a critical criterion is that the fracture process zone (FPZ) where nonlinear material response is occurring, must be confined to an area around the crack tip that is small compared to the fracture length and other dimensions of the system. Assessment of this criteria requires a methodology for estimating the length of the FPZ. To make this assessment and for cases where this assumption is violated, nonlinear fracture mechanics methods must be devised and employed.

3. NONLINEAR FRACTURE MECHANICS MODEL FOR ROCK

A relatively simple interface model for the FPZ has previously been developed and applied extensively for the tensile failure of rock based on the assumption that rock can be characterized as a strain-softening material. The bulk of the rock mass is assumed to behave elastically, and nonlinear deformation is assumed to be confined to a narrow fracture layer as shown in Figure 2 (Boone et al. 1986). The FPZ starts at the point, where the nonlinear strain-softening begins, and extends along the fracture to where the crack faces have zero tensile strength. A key concept in the rock interface model is that a simple tensile test can be conducted where the stress versus displacement in the narrow region of nonlinear behavior can be characterized as shown in Figure 2. Furthermore, using principles associated with the J-Integral technique and assuming self-similar fracture propagation, it can be shown that the Critical Fracture Energy (G_c) of the material is equivalent to the area under the material failure curve shown in Figure 2. By definition, G_c is a material property that can be employed in LEFM analyses when the FPZ length is small compared to all other dimensions of the system including the fracture length. However, when explicitly applying the full stress distribution of the nonlinear fracture model, there is no limitation on the dimensions relative to the FPZ.

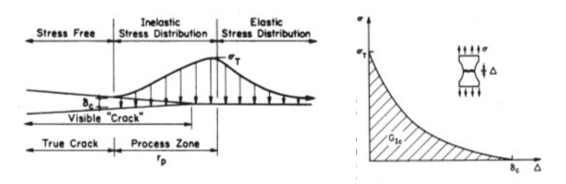


Figure 2. Nonlinear interface model for fracture in rock (Boone et al. 1986)

This interface model was developed for tensile fractures, which are the most common concern in rock mechanics. In the sections that follow, the model is adapted for compressive failures or anticracks within weak layers in snow packs.

4. EXTENSION OF THE INTERFACE MODEL TO WEAK LAYERS IN SNOW

A typical weak layer in snow results from hoar frost or other similar events that lead to the formation of a layer of ice or snow with a high porosity or low density and a strength that exceeds that required to support the overlying snow pack. As such, the term 'weak layer' is somewhat of a misnomer. Weak layers have a relatively high strength, given their density, that results from bonding between crystals of snow or ice. The critical feature of a weak layer is that when it initially fails and the bonds between the ice crystals break, the density of the layer increases while its load bearing capacity decreases (i.e. strain-softening).

It can be hypothesized that a simple compression test on a weak layer in snow will have the form shown in Figure 3 where three regions are identified: elastic loading, strain-softening and compaction. In order to capture the strain-softening region of the curve, the test must be conducted by means of controlled displacement rather than load or stress control. For simplicity, the paper considers failure of a flat lying weak layer with a 0-degree slope. The methods can be generalized to angled slopes but, as noted previously, unstable failure propagation in weak layers do occur in flat lying snow packs and are observed as whumpfs.

The interface model for tensile failure in rock considers only the post failure strain-softening zone since the crack or narrow zone of nonlinear behavior is assumed to have zero thickness, and the rock faces are assumed to completely separate. For the weak layer in snow, the model allows for elastic deformation across the finite thickness of the weak layer, inelastic deformation of the weak

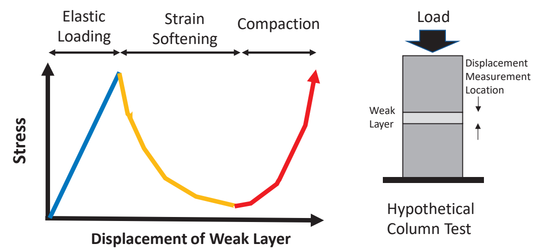


Figure 3. Displacement controlled compression test of a weak layer in a snowpack.

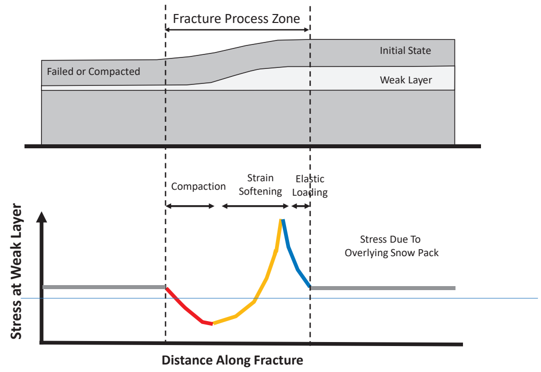


Figure 4. Nonlinear interface model adapted to compressive failure of a weak layer in a snowpack.

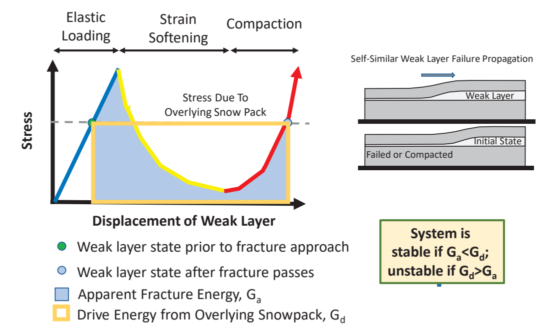


Figure 5. Stability assessment utilizing the weak layer compression test.

layer through strain-softening and subsequent strain hardening or compaction of the ice crystals.

The resulting stress versus displacement response for the weak layer that is shown in Figure 3 is applied as a nonlinear interface model to partially failed weak layer in a snow pack as shown in Figure 4. Ahead of the FPZ, the weak layer only sees the stress of the overlying snow pack. The leading region of the FPZ is the elastic zone where there is deformation in the weak layer due to the stress concentration. Immediately behind the elastic zone is the strain-softening zone where the stress in the weak layer is declining from the peak compressive strength. Further back is the strain hardening zone where the stress on the layer increases until the full load of the overlying snow pack is supported by the compacted weak layer. In this case, the length of the FPZ is defined as the combined length of the elastically compressed, strain-softening and compaction zones.

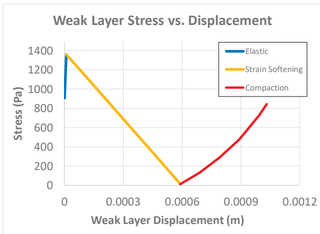
Assuming self-similar fracture propagation, the material resistance to fracture propagation or apparent fracture energy, G_a, can be determined from the area identified in Figure 5. Note that this linkage between the results of a compressive test and a propagating fracture is a key relationship that makes this model an especially simple and useful tool. In most cases, it is reasonable to expect that the assumption of self-similar fracture propagation is valid once the failure has propagated a short distance away from the failure initiation point. However, since the area identified as the fracture energy is influenced by the thickness and density of the overlying snow pack, it is an apparent Fracture Energy, G_a. The term Critical Fracture Energy, Gc, is reserved for true material properties that are independent of geometry.

The primary drive energy for fracture propagation (away from any localized load such as a skier) is the weight of the overlying snow pack. Energy released or drive energy (G_d) for the fracture propagation is due to the downward displacement of the mass of the overburden. G_d is also shown graphically as an area in Figure 5. A key result from the model is that if $G_d < G_a$ the snow pack is stable and, if $G_d > G_a$ then the snow pack is unstable. Local failure of the weak layer can occur for a variety of reasons such as a skier passing over the area, a falling cornice or localized snow build-up such that the compressive strength of the weak layer is exceeded. However, *a necessary condition for failure of the weak layer to propagate 'unstably' over a large area, and potentially initiate an avalanche, is $G_d > G_a$.*

It should also be noted that similar low porosity weak layers are found naturally in rock and have been studied extensively as well (Sternloff et al.)

5. QUANTIFIED PARAMETERS FOR THE MODEL

The model can be quantified and assessed using material properties available in the literature. An example set of data is included in the table in Figure 6. Jamieson (1995) provides averaged data for a set of skier-triggered avalanches which has been used for the overlying slab thickness, its average density, the weak zone thickness and its density. The peak strength of the weak zone is assumed to 150% of the induced stress on the weak layer. The fracture energy associated with the strain-softening is assumed to be approximately equal to the mean fracture energy for weak layers measured by van Herwijnen et al. (2016). Given the fracture energy, and the peak strength of the weak zone a linear slope for the strain-softening portion of the stress strain curve has been calculated. Jamieson (1995) also provides a set of measurements of snow slab density versus depth which can be converted into density versus stress which has been used to derive the equation for the compaction portion of the curve. The resulting plot for the stress-displacement is shown in Figure 6. Comparing the drive energy, G_d, with the apparent fracture energy, G_a, it can



Model Parameters	Values
Overlying Snowpack	
Thickness (m)	0.57
Average density (kg/m^3)	162
Weak Layer	
Thickness (m)	0.0021
Average density (kg/m^3)	121
Axial Elastic Stiffness (Pa)	1.00E+05
Peak Strength (Pa)	1358.7831
Fracture Energy (J/m^2)	4.00E-01
Compaction Stress (Pa)	= A + B * Density^2
A =	-819.1
B =	0.02937

Figure 6. Quantification of the stability assessment utilizing the weak layer compression test.

be seen that G_d>G_a which indicates that the system is unstable. This is consistent with the data set being sampled from field cases where skiers had induced slab avalanches.

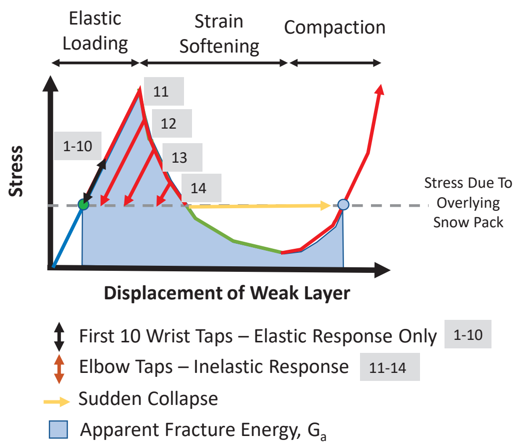


Figure 7. Interpretation of the field compression test.

6. INTERPRETATION OF THE FIELD COMPRESSION TEST

When considering a standard laboratory tension or compression test for materials, one typically controls the load while measuring corresponding displacement until the point of failure. This works well for strain hardening materials such as steel. For strain-softening materials, one needs to control the displacement and measure the load to capture the full stress displacement curve as shown in Figures 3 and 5. Neither of these models applies to the field compression test for snow, as described by Jamieson and Johnston, 1996, where a column of snow is loaded to failure through a series of wrist taps, elbow taps and then full arm taps. It is more appropriate to consider this test as an energy or damage related test where, if a tap is sufficiently forceful to exceed the strength of the weak layer, each tap contributes a unit of energy towards the failure of the snow pack. Figure 7 provides an example of how the test may be analyzed in this manner.

Assuming that the snow overlying and underlying the weak layer behaves elastically and all the nonlinear behavior is concentrated in the weak layer, Figure 7 shows a case where there is a ‘sudden collapse’ on the 14th tap in a field test. The first ten “wrist” taps generate a peak compressive stress that is less than the compressive strength of the weak layer and do not result in any nonlinear deformation of the layer. The higher stress generated by the subsequent elbow taps is sufficient to exceed the compressive strength of the layer and results in progressive damage. Assuming an approximately equal energy is imparted by each tap, the stress displacement curves for each elbow tap are traced in Figure 9. The area covered by each stress-displacement cycle corresponds physically to progressive breakage of bonds between the snow crystals and rearrangement of the snow crystals in the weak layer.

In the example, on the fourth elbow tap a sudden collapse or jump in the displacement occurs. This event occurs because the incremental rate of energy absorption of the weak layer due to additional displacement is less than the rate of energy released due to the incremental downward displacement of the overlying snow pack.

It is important to note that this sudden catastrophic failure during the test does not necessarily imply an unstable condition in the snow pack. The snow pack is unstable if the net energy released after the point of sudden catastrophic failure is greater than that absorbed by the weak layer due to the previous taps as per the criterion in Figure 5. However, common field guidance is that a sudden collapse in the weak layer during a compression test is always a cause for concern. Based on this interpretation method, it seems to be very reasonable guidance as it is difficult to quantify the relative energy absorbed during the taps compared to that released during sudden collapse. Nonetheless, a secondary consideration can be the number of taps at a given stage (wrist, elbow or arm) required to initiate sudden collapse.

A limitation of the field compression test relates to the assumption that the overlying snow pack responds elastically. Often the overlying snow pack is composed of loosely packed snow that is inelastically compacted during the test. This compaction will absorb energy from the taps and may in effect shield the weak layer from the imparted energy and progressive damage. During the process of unstable propagating failure in the weak layer, it would not be expected that the same inelastic deformation would occur in the overlying snow pack so it should be considered an artifact of the field test. Arguably, it could be better to remove any very low density weakly packed snow from the top of the snow column. However, this will remove a fraction of the natural drive energy due to the weight of the overlying snow pack. A more practical and conservative approach would be not to count taps that induce significant compaction of the snow column above the weak layer.

7. PROPAGATION SAW TEST

The Propagation Saw Test involves isolating a block of snow, creating a saw cut long the weak layer and monitoring for unstable propagation along the weak layer, Gauthier et al. 2008. If unstable propagation occurs, the failure in the weak layer may rapidly progress through the full block or it may arrest within the block.

The most apparent limitation of the test is that a saw is used to disrupt the weak layer so that at the point of unstable failure propagation, the region behind the leading edge of the saw has properties that are not representative of the failed weak layer elsewhere.

Detailed analyses of Propagation Saw Tests which have included multiple displacement measures, an estimation of the stiffness of the snow pack and deformations of the block behind the saw cut may allow for reasonable estimates of the apparent fracture energy of the weak layer. However, given the challenges cited above, any interpretation of snow pack stability from simpler field tests using the Propagation Saw Test are not likely to be reliable.

8. NUMERICAL MODELING

A number of finite element and discrete element codes include interface and contact models that allow for implementation of the weak layer model described in this paper. Many have been used to successfully model analogous nonlinear fracture mechanisms in rock (e.g. Detournay et al. 2003, Boone et al. 1991). These same capabilities can be readily adapted to mechanics of weak layer failure and slab avalanches.

9. SUMMARY AND CONCLUSIONS

This paper provides an adaptation of a relatively simple nonlinear fracture mechanics model for rock to mechanics of weak layer failure and propagation in snow packs where the weak layer failure and propagation is a key step in the initiation of slab avalanches. Most importantly, the model provides a critical linkage between the stress-displacement response of the weak layer and stability of the snowpack by relating the drive energy, G_d, released by the gravity drive displacement of the overlying snow pack to the energy absorbed by the material failure in the weak layer, G_a. The weak layer is unstable if $G_d > G_a$ meaning that failure, once initiated, will continue to propagate.

The linkage between stability of the snowpack and the stress-displacement response of the weak layer allows for a simple interpretation of the field compression test in terms of damage or energy absorption by the weak layer as illustrated in Figure 7. Application of the model should enable improvements in the interpretation of field compression tests. On the other hand, the model highlights the challenges of the Propagation Saw Tests due the disturbance of the weak layer by the saw cut and the impact of the stiffness of the overlying slab.

The features of this model have already been incorporated into a number of commercial finite element and discrete element codes. There is considerable potential for the snow science community to further adapt and apply the same methods and tools.

* *Corresponding author address:*
Tom Boone, Ph.D., P.Eng.
tel: +1 587-830-3759;
email: tjboone27@gmail.com

ACKNOWLEDGEMENT

Many thanks to numerous guides and teachers of snow safety as well as Tony Ingraffea, Emmanuel Detournay and Renee Boone for reviewing the paper!

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